NCE-003-1162003   Seat No. _____

M. Sc. (Sem. II) (CBCS) Examination

April / May - 2017

Mathematics - 2003

(Topology - II)

(New Course)

Faculty Code : 003

Subject Code : 1162003

Time : $2\frac{1}{2}$ Hours]   [Total Marks : **70**

Instructions : (1) There are five questions in this paper.

(2) Each question carries 14 marks.

(3) All questions are compulsory.

1 Fill in the blanks: (Each question carries two marks)

(a) In a T_1 - space every singleton subset is _____.

(b) Every compact subspace of a Hausdorff space is _____.

(c) Every Locally compact Hausdorff space is _____.

(d) A complete subspace of any metric space is _____.

(e) Urysohn's Lemma is equivalent to the separation axion _____.

(f) A closed and bounded subset of $\mathbb{R}^n$ is _____.

(g) The one point compactification of a non compact Locally compact Hausdorff space is _____ and _____.

2 Attempt any two of the following :

(a) Prove that any open subspace of a Locally compact   **7**
       Hausdorff space is Locally compact.

(b) Prove that   **7**

(i) Every subspace of a Hausdorff is Hausdorff.

(ii) Suppose X is a T_1 space which satisfies Uryson's Lemma. Prove that X must be normal.

(c) Prove that every compact space is limit point compact. **7**

3   All are compulsory :

(a)   Prove that any compact subset of a Hausdorff space is   **6**
closed.

(b)   Prove that any closed subspace of a complete metric   **4**
space is complete.

(c)   Prove that a T_1 space X is regular if and only if for   **4**
every open set U and $x \in U$ there is an open set V such
that $x \in V \subset \bar{V} \subset U$.

OR

3   All are compulsory :

(a)   State and Prove Lesbesgue's Covering Lemma.   **7**

(b)   Prove that a T_1 space X is normal if and only if for   **4**
each closed set A and an open set U with $A \subset U$ there
is a closed set V such that $A \subset V \subset \bar{V} \subset U$

(c)   Give an example of an infinite topological space which   **3**
is not compact.

4   Attempt any two of the following :

(a)   State Tube Lemma and then prove that $X \times Y$ is   **7**
compact if both X and Y are compact.

(b)   Prove that Y^X with uniform metric is a complete metric   **7**
space if Y is a complete metric space.

(c)   Prove that $\mathbb{R}^n$ is a complete metric space.   **7**

5   Do as directed : (Each question carries two marks)

(a)   Give the definition of a family of subsets having Finite
intersection property.

(b)   Give the two subsets of $\mathbb{R}$ (the set of real numbers with
standard topology) such that one is closed but not
bounded and the other is bounded but not closed.

(c) Let $A = \left\{ 1, \dfrac{1}{2}, \dfrac{1}{3}, \ldots\ldots\ldots, \dfrac{1}{n}\ldots\ldots \right\} \cup \{0\}$. Is A a compact subset

of $\mathbb{R}$? Give reasons for your answer.

(d) Give an example of an Uncountable complete metric space which is a proper subspace of $\mathbb{R}$.

(e) Define : (i) T_1 space and (ii) Completely regular space.

(f) Give an example of a countable subset of $\mathbb{R}$ which is not Locally compact.

(g) Determine if the set [0, 1) is a complete subspace of $\mathbb{R}$ or not ?

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